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ABSTRACT

A cognition theory of learning is described based on information processing analyses of representations of knowledge in long term, intermediate term, and short term memory. An application of the theory is presented for an example of algebra. It is emphasized that learning is not simply a stamping in, through repetition, of simple links. The associations are complex. Some are related to our use of language; verbal instructions are an important means by which a teacher communicates. Some of the associations involve actions that have perceptual and motor components. By gaining control of the instantaneous attention of the student, we are in a position to directly influence his construction of knowledge. This is the goal of a cognitive theory of learning. (Author/RC)

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Content and Structure in Learning

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By the time a student is ready to apply the quadratic formula to a problem like $5x + x^2 = -6$, he has, as the expression goes, forgotten more about number lines than many of us will ever know. Much of the specific learning in grades 2 or 3 that went into the development of his current collection of mathematical wisdom has been lost by grades 9 or 10. He has committed to long term memory multiplication tables, the basic operations of addition, subtraction, and division. He may still have some trouble with the notation system whereby the variable x stands for some indefinite quantity. He may also ask why learn this formula, since there is no immediate application of it. Perhaps only later, in analytic geometry, will the relationships take on visual or spatial meaning to him. He will, however, have some very direct and recent information about the roots of the equation and factoring.

$$(x + 2)(x + 3) = x^2 + 5x + 6 \quad (1)$$

He will be struggling to discover the relationship between factoring and the quadratic formula.

$$x + \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad (2)$$

What does it mean to say that a student has learned the quadratic formula? Is it when he can reproduce, after a period of time, those symbols in written form? Is it when he can produce the verbal symbolic equivalent, "x equals minus b plus or minus the square root of b-squared minus 4 a c, all over 2 a." We would additionally expect the student to demonstrate his knowledge of the quadratic formula by solving some problems. Associated with the symbolic content of the formula are some rules for its application. Where do the values of a, b, and c come from? Each problem must be put into canonical form. The basic arithmetic operations must be carried out correctly. In order to map problem onto a formula and to perform the steps in its solution, the student must have a specific plan for executing the sequence of steps. He has to keep track of his place. He has to remember intermediate products of his calculations. The short term memory, which we will here call the immediate processor, is not an automatic, guaranteed, all-purpose calculator. The student must learn to manage the short-term memory limitations of this bottle-neck in the human information processing system.

Three Domains of Memory

What we mean by content and structure in learning is essentially the three domains implied by the kinds of information that must be in the student's memory, as described in the example. The first domain deals with the representation of knowledge in long-term memory (LTM). The second involves the acquisition of specific sequences of skilled performance, in what we will call working memory; and the third, the control of attention in short term or immediate memory.

Representation of Knowledge in LTM

Recently there has been a great deal of interest in psychology and in artificial intelligence in the way to represent ideas and concepts in long term semantic memory. One result of the computer revolution is that more psychologists than care to admit it have become associationists. Anderson and Bower (1974) develop the point, arguing that the only way new information can get into the memory system is to link it to old information already present in the memory.

Although there is no conceivable way to test directly the precise form of internation representations, in the abstract it is possible to show relationships by means of node-linked memory diagrams. These are directed graphs where the nodes are undefined, the links are relations, and primitive

 Insert Figure 1 about here

elements have specific experiential counterparts. Links specify the types of relations between nodes in the semantic memory. For us, nodes roughly correspond to concepts in the long term memory system. Many of these, certainly the ones we can talk about, are word concepts and hence have symbolic names, words, in language. In general, most of the relations are based on an analysis of the logical structure of language and of naturalistic taxonomies. Most of these representations of knowledge are tied to verbs, which drive the representation of meaning.

The very long term memory is closely related to the kinds of formalization that our language allows, and the content is that set of word

concepts, organized in terms of semantic relations. The content of long term memory is wisdom. This representation of knowledge comprises the bulk of the information available for problem solving and intentional learning. After all, this is the accessible information that we have.

At a later time, the student may ask, "Let me see, can I remember the quadratic formula?" If there is a node in the long term memory structure, and if either the orthographic generator or phonemic generator remains intact, then that formula can be retrieved and used.

When we acquire specific cognitive skills, concrete representations of information are put together. When the student commits to memory that formula, when he does his homework, applying the formula to individual problems, when he "remembers" the answer, when he recalls the formula under the particular circumstances of a midterm examination; each of these specific experiences with the quadratic formula provides an opportunity for establishing meaningful, i.e., semantic, relations to already formed cognitive structures. Linguists such as Eve Clark (1973) describe the growth of full word concepts as the acquisition of semantic features. Just what these may be, in the context of the quadratic formula, is not clear. However, we may speculate that one semantic element must be the relation between the letters in the formula, a , b , and c , and the coefficients, or, more precisely, their positions in the canonical form of the equation. Only when this is understood can the formula be applied to an arbitrary equation.

Another semantic element that makes sense of the "plus or minus" sign is the realization that the quadratic formula has two roots. Only in the

case of perfect squares do these answers turn out to be the same value. All the learning that has gone before, much of the experience that will come later, determine the "real" meaning of the quadratic formula.

What is the minimum set of semantic relationships that is necessary before we "know" the quadratic formula? In some respects this is a foolish question, because obviously there is no end to the depth of understanding that is possible. Learning is a continuing process of adding new relations and revising old ones. It is probably the case that when the specific concrete experiences that led to the concept are themselves forgotten, the formula leaves the domain of specific learning, and becomes a part of the more general semantic structure. However, in order to use the quadratic formula to solve problems, the student must learn a specific sequence of steps and be able to carry them out.

Specific learning in the case of the quadratic formula involves building up the computational sequence. Knowing what to do next in performing the arithmetic operation can be quite complex. Such a sequence may include

 Insert Figure 2 about here

the following steps. (1) Can the quadratic formula be applied to this equation? (2) If yes, then is the equation in canonical form? (3) If yes, get values of the coefficients a , b , and c . (4) Calculate discriminant functions. (5) If imaginary, execute alternate plan, and stop. (6) Calculate value of radical. (7) Get numerator plus, calculate denominator and divide to get root 1. (8) Get numerator minus, remember denominator, divide to get root 2. (9) Stop.

For those who have hand calculators, the instruction book specifies an order in which operations are to be performed, to avoid writing down intermediate products, if at all possible. To the extent that those programmatic sequences are learned, practiced to the point of habitually correct performance, an important aspect of the concept has been learned.

Specific skill learning is an ordered sequence of behaviors. Such a program specifies and controls our attention to the objects of learning, serves to retrieve appropriate operators for the transformation of those objects, and provides the feasibility tests for applying those operators. When that sequence becomes automatic as a consequence of repetition, then cognitive control of the program decreases. This is both an advantage and a disadvantage. If you don't have to think about all the steps in the sequence, and check what to do at each stage of its execution, you can perform the task faster. However, the opportunity for error, undetected error, increases.

But the fact is that those automatic performance programs become the functional or action parts of our concepts, by speeding up the process of obtaining results, by chunking, integrating behavior. It becomes possible to turn our attention to other aspects of the task. It makes it possible for us to detect the relationship that the product, " $4a^2$ " is greater than " b^2 ". We have an imaginary square root, and hence the result is a complex number. These are relationships that you would not otherwise have noted, because of the limited capacity of the information processing system.

The Limited Capacity of Immediate Memory

What kind of information gets into the human short term memory? The

first kind of information is modality-specific, coming from the visual or auditory stores. We believe that there are a number of memory systems associated with sensory pathways, and that sensory information intersects with the portion of long term memory that has to do with recognition of objects. We believe that the essential nature of this memory system is a tree of discriminative tests. We believe that there are fundamental timing cycles of the brain such that both visual and auditory signals generate images that can be denoted symbolically in short-term memory. However, the short term memory has a limited symbolic capacity. There are several ways to overcome the limitation of immediate memory. Depending upon the task at hand, various strategies can be developed that employ rehearsal, mnemonic devices, recoding, and chunking. Consider rehearsal. A rehearsal strategy is simply locating a subset of the elements in the auditory short term memory, and recycling them one at a time. In rehearsal, timing parameters are important. These depend on the basic rates of producing phonetic (speech) patterns. Rehearsal is a strategy for increasing the apparent capacity of the short term memory by using a property of a sensory system. One uses the sensory system with the longest half-life for information to maximize the amount of information that stays around. Therefore, visual events get named, phonetically encoded, and the names get cycled in the auditory system. Under ordinary circumstances, most visual images will disappear in a quarter of a second (Sperling, 1960), but by retrieving the name of the object and saying it to oneself, the capacity of short term memory is increased. Data from Conrad's (1971) studies of acoustic confusions indicate that children begin to exhibit this strategy around age 5.

Other methods make use of mnemonic devices and chunking. Both of these depend on long term memory, and prior programmatic learning of specific data structures.

Finally, there are strategies for managing short term memory that depend on temporal organization of the processing sequence. They capitalize on the fine timing capacities of the system. By synchronizing behavior with external or internal events, creating rhythmic patterns which can serve as cues, skilled performance that seems to demand great short term memory capacity can be achieved.

In summary, the human cognitive system deals with symbols that are organized at three levels of complexity. The operations that can be performed in relation to these symbols vary in terms of their computational power and speed. For example, in the Immediate Processor, the operations are restricted to scanning either in forward or reverse order, and very little else. But the scanning rates are rapid, according to some experiments performed by Sternberg (1969), less than 50 milliseconds per element. Programmatic learning skills, however, seldom exhibit operations in less than 100 milliseconds, and decision processes that are involved in the integration of linear sequences may exceed a second or more. Long term memory is complex, in that there is an elaborate pattern of search--heuristic search. The programs that operate here are general problem solving programs, and the speed of operation is of the order of 1 or 2 seconds, or more.

In the next section, we will address the domain which is most familiar to instructors--the domain of specific skill learning. This type of learning occurs in the context of well-defined educational objectives.

These are the behaviors that teachers set out to teach. They may be as extensive as a 4-year sequence of course in Russian, or as limited as learning how to use a hand calculator to do the quadratic equation.

Specific Skill Learning

First, let us consider the ways in which specific skill learning is affected by the contents and structure of the basic knowledge domain. Since the content of long term memory is word concepts, it is through language communication that the teacher can give instructions for tasks that the learner must perform. What is involved in understanding instructions? In comprehension of the written or spoken instructions, the learner is using language skills, to be sure, but more importantly--he is carrying out a problem-solving task. He is trying to figure out what the referents of the words are in terms of his immediate surroundings; he is establishing priorities for attention among the collections of objects in the learning situation; he is building up a workable representation of what it is he is expected to do. Thus, the first problem of the learner is to understand what is being asked of him. Clearly, rich semantic relations are more likely to permit the discovery of meaning.

The way that this occurs is through the processing of language. But we are not talking about a simple grammatical analysis in which the instructions are parsed sentence-by-sentence to identify agents (nouns), actions (verbs) or objects, per se. Most formulas are expressions involving numerators and denominators. So the word concept "one thing to be divided by the other" is a general property of all formulas. The sets of relations that tie nodes in the semantic memory together provide a

general framework onto which new learning can be grafted. When a student is faced with the specific learning of the quadratic formula, he must pay attention to the fact that the term is a way of getting into the memory structure--i.e., the name of a thing is an index to information about the thing. It is extremely important in specific skill learning that the index remains stable. Very often a teacher, in trying to elaborate examples of the point, creates multiple indices--not necessarily to the same information--and the student becomes confused. Thus, indiscriminant use of the words "equation" or "formula" may lead to confusions during specific skill acquisition. A quadrative equation is the expression to which the formula is applied to solve for x . Obviously the fact that the formula has an equals sign in it makes it an equation as well. The point is that care in labeling is an important prerequisite of building retrieval cues for information in the semantic net work. Those previously learned properties of formulas are what we hang the new information on. "What do I know about formulas? They let me calculate answers. Therefore I must substitute values into them." If, on the other hand, the student is thinking of equation, he may become confused. Equations are, in effect, the questions. Formulas are the answers. And what we mean by the phrase "talking over the student's head" is that a more sophisticated teacher may use a word expression to stand for the class of both formulas and equations, not realizing that the new word points to different information.

The long term memory must also have executable programs, and these guide the assembly of specific skill components. The knowledge that formulas have numerators and denominators directs the student's attention

to those parts of the formula. When the teacher says, "In the numerator" the mind's eye literally turns to a place. It is not simply that a word means a certain part of a formula, explicit actions are dictated by that knowledge.

Teachers in the short run have very little control over what goes into the semantic memory. It is not easy to teach wisdom. Perhaps we should, however, pay more attention to the teaching of heuristics and general problem solving procedures. Too often we teach just content, just the names of things, and not the relations among them.

Management of Memory and the Control of Attention

The immediate memory limits the rate of acquisition of new information, and is itself subject to limited control. There is not a great deal we can do to change its properties. The methods suggested above, for managing short term memory--rehearsal, chunking, mnemonics, and temporal organization--are not powerful factors. For example, there are practically no effects greater than two-to-one differences on any of the parameters obtained in the usual short-term memory experiments. Memory span is seven, plus or minus two (Miller, 1956); that is a 1.80 to 1.00 ratio. The number of variables that one can keep track of is two or three (Yntema & Musser, 1960, 1962); that is a 1.5 to 1.0 ratio. Scanning rates are but a small part of the total time to search short term memory. At 50 milliseconds per item, the additional 300 milliseconds for six items must be added to a base reaction time of about 400 milliseconds, so that even as scanning rates approach 0 milliseconds, this is only a 700 to 400 ratio, or 1.75 to 1.00 difference. Implicit rehearsal ranges from 3 for the

laggards to 5 symbols per second - 1.67 to 1. Other parameters, such as duration, access time, and storage time, show similarly small individual differences.

However, there are obvious differences in academic performance that exceed 100 to 1. What are they? The Class A chess player from a club in the United States will never win a single game from an international Grand Master (Simon & Chase, 1973). The problem solving capabilities of near genius students are entirely beyond the scope of dull normal ones. We have shown that the major circumstances that account for these differences are attributable to long term memory, and not the fine tuning of the short term memory.

We cannot train the short term memory to do much more than it is capable of doing. The immediate memory imposes a limitation. However, it is a very important limitation that the instructor must cope with. The opportunity for learning depends on having symbolic content available in the short-term memory, and having it there in usable form. Now what do I mean by all of this? I'm saying that the elements of a formula must be available long enough for relationships among them to be discovered, labels to be attached, and appropriate hooks in the long term memory to be found. Forming a simple association is in fact a formidable juggling act wherein new symbolic elements and old ones must be available to the immediate, conscious, cognitive system.

How can we accomplish the juggling act? Obviously good visual aids provide an external memory for the elements. Writing the formula on the blackboard is very important. Secondly, some way must be found to direct

attention to the separate elements that must be hooked together. For example, when I earlier mentioned the imaginary numbers, the student's attention must be directed to the part of the formula under the radical. Then, the quantity $4ac$, since it is negative, is juxtaposed--i.e., put to the left of the b -squared, and the symbolic operator "greater than", interposed. Notice that we have gone from one representation--" b square minus $4ac$ is a negative number"--to an alternative description--" $4ac$ is greater than b -squared." The meanings are very similar, but the latter formulation states a criterion, a test for determining if the equation has real roots, and implies a further semantic feature.

Incidentally, computer programs written to solve the quadratic and similar formulas first check to find out if the roots are real or not. This is because the efficiency of processing by computer can be increased by knowing this fact. The typical novice algebraicist, on the other hand, is usually startled when he notices that he is about to take the square root of a negative number.

Over the years, our experiments and research on simple forms of association learning have consistently shown that the collection of micro-processes--the juggling act I referred to above--consume on the order of 10 seconds per association (Gregg, 1972). Such a detailed level of description may be more useful in the classroom than you think. If we can deal with six events per minute, and there are 50-minute class hours, then 300 significant events will have occurred during any given class. What are they? Do you know? Have you planned them? Are they word associations? Unfortunately, the entire psychological literature on human verbal learning

has very little to say about classroom learning, since the repeated trials and 2-sec presentation rates are unrealistic simulations.

Cognitive Theory of Specific Skill Learning

So far we have considered some aspects of the influence of long term memory on SSL, and the constraints imposed by short term memory on SSL, and now we are ready to address the question of specific skill learning itself. We have hinted at some of the obvious processing components. Such principles as: temporal contiguity--you've got to get it together; the rehearsal principle--you've got to keep it around; the principle of indexing--you've got to get it back; and the "act of discovery"--you've got to find new relationships.

Such disparate ideas really do not constitute a theory. They simply are statements about the learning process equivalent to, say, the empirical law of effect, which, incidentally, is an entirely satisfactory formulation of the motivational aspects of learning. Reinforcement, reward, satisfaction with one's behavior, keeps it going. But what we are really talking about in a cognitive theory of learning is a much more detailed description of the ongoing processing of information.

Specific skills are learned in delimited task environments, for which we can specify three things: (1) the content in terms of an external task environment (2) symbolic representation of that content; and (3) a strategy that describes the management of the immediate memory. This amounts to an instructional task analysis (Gregg, 1975).

The first step is the description, from the viewpoint of the instructor, of the external task environment. In other words, the collection

of formulas in algebra and operations from the teacher's point of view, which are to be learned. This is necessary because, first, appropriate evaluation measures must be devised to see how close the student is coming to the objectives. And second, the teacher must be able to assess where the student is at any given time. The instructor's representation of the external task environment must specify content and structure--that is, elements and relationships among them--and performance requirements as well.

The second step is, on the basis of initial performance and test outcomes, the teacher must model the internal representation of the semantic memory of the student. Content and structure refer to the knowledge that the student can bring to bear. Of course, every teacher develops the ability to communicate with a student. If you think hard about this in terms of the task at hand, what you are assessing is the richness of the semantic network, the extent and kinds of relationships of Figure 1 that can be drawn upon.

Finally, the third aspect concerns the specific learning strategy, the sequence of behaviors governing performance and exerting control over attention. Where attention means the selective and varying focus on aspects of the task. The example that we had earlier was paying attention to the $4ac$ before the b^2 , even though the b^2 was physically present in the formula to the left of the $4ac$. If you had a bouncing ball, where would it bounce? Where would you like to have it bounce?

On the basis of such an instructional task analysis, the teacher must find out what the student knows, what he can do and match the instructional sequence to those capabilities.

Teachers have been examining what students know for a long time by asking questions like the following: What is the equation for a parabola? What is the discriminant function? How many roots does a quadratic equation have? What is an imaginary number? Define factoring. Each of these questions examines the content of long term memory and the ability of the student to retrieve verbal descriptions associated with the concepts. Notice, however, that very little in the way of structure can be inferred from correct or incorrect answers to these questions. A somewhat different technique has been used successfully in a few of my classes to gauge not only the sophistication but the organization of student memory. Rather than asking for definitions of individual items, or assigning essays to be done at home where revisions are possible, I have asked students to free associate about the "concepts you learned last semester in such-and-such a course." A content analysis of the clusters exhibited by students who had formal training in the course, as compared to students who had not, showed that sophisticated students produced 4 items per cluster on the average, against the naive students' 1.5 items. For example, in the psychology of learning, the naive students would produce lists like the following: reinforcement, motivation, then conditioning - Skinner... Only one or two items on the average would be considered members of the same superordinate category. A sophisticated subject, mentioning classical conditioning, would then list items such as extinction, generalization, unconditioned stimulus, conditioned response, temporal interval, and so forth--giving evidence of coherent organization in long term memory.

To find out what the student can do, teachers give such problems as "What's the square root of 25?" Is that a skill, or is that knowledge?

If it is a table look-up, obviously we're not talking about skills, but about knowledge. On the other hand, calculating the square root of 1069, or some other novel stimulus, demonstrates that the student can perform the necessary sequence of calculations. Asking the student to solve by factoring, to derive the quadratic formula, to apply the formula in solving a given equation, are ways of testing his computational proficiency. One problem, however, is that strict hierarchies of skills do not exist. This is the same problem that linguists encounter when they discover that a child can learn the class name tree before learning subordinate members of the class such as oak or maple. The structure of semantic does not conform to the neat taxonomies that hindsight and formalism can provide. How then is the teacher to develop reasonable methods of assessing these skills? Most of the time we do not use formal methods at all, but simply begin teaching, and adjust our methods to take account of what the students are able to learn. Most teachers simply emphasize the content of learning, and ignore the skill components. We teach what you should know, not how you should learn.

Nor do we teach heuristics for managing attention. Attention to many teachers means only, "Is the student sitting up straight and looking toward the front of the room?" Most of us never think about the way the student's mind rapidly shifts from perceptual acts, to problem solving processes, to skilled performance. If a quadratic equation is presented, Figure 3 illustrates how perception, planning, and the recall of skills

 Insert Figure 3 about here

might appear sequentially to control the cognitive behavior during the student's solution of the problem.

In this analysis of content and structure in learning, we have attempted to show how 3 levels of information interact during the learning process. It is important to realize that learning is not simply a stamping in, through repetition, of simple links. The associations are complex. Some are related to our use of language; verbal instructions are an important means by which a teacher communicates. Some of the associations involve actions that have perceptual and motor components. By gaining control of the instantaneous attention of the student, we are in a position to directly influence his construction of knowledge. This is the goal of a cognitive theory of learning.

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Footnotes

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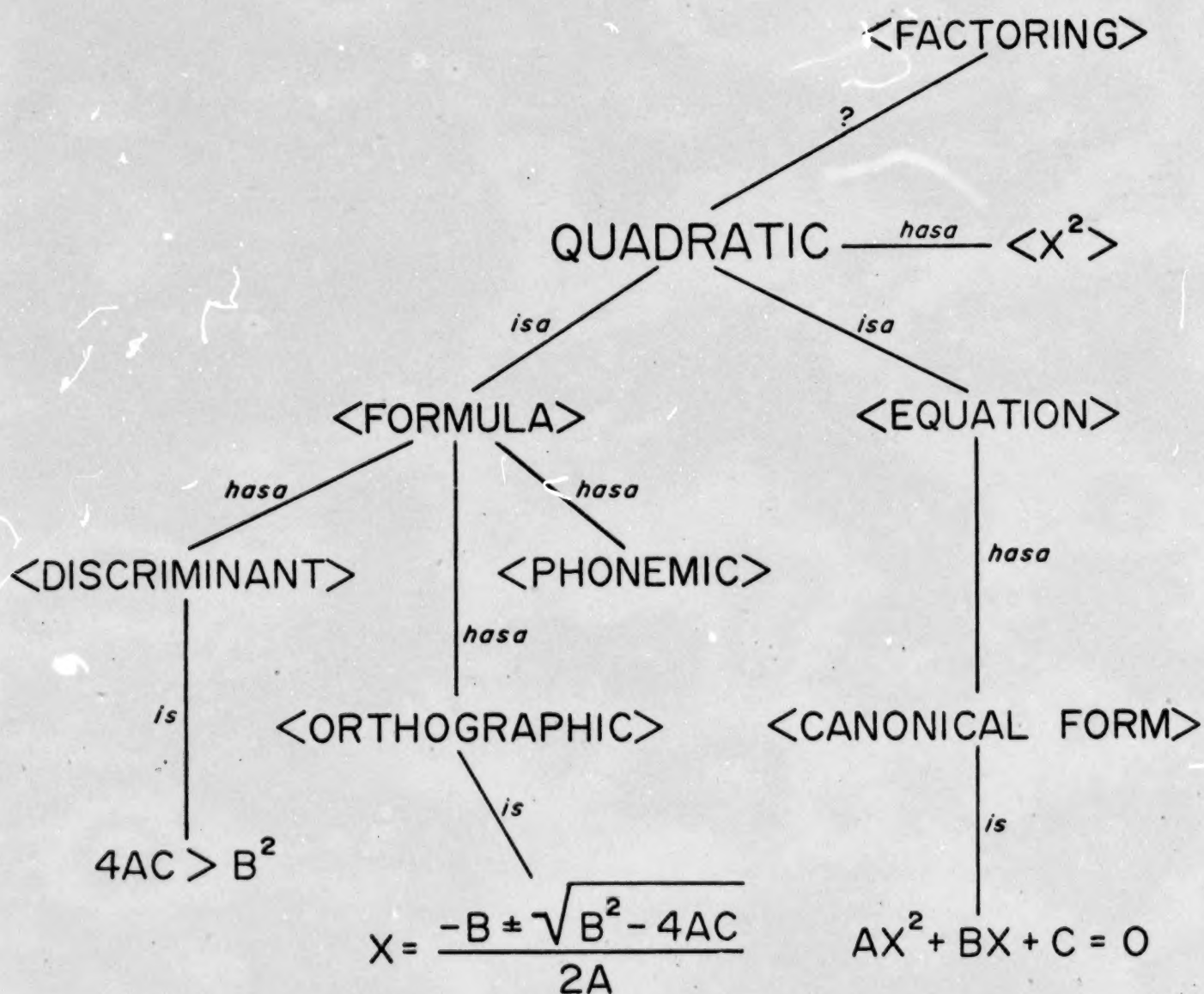
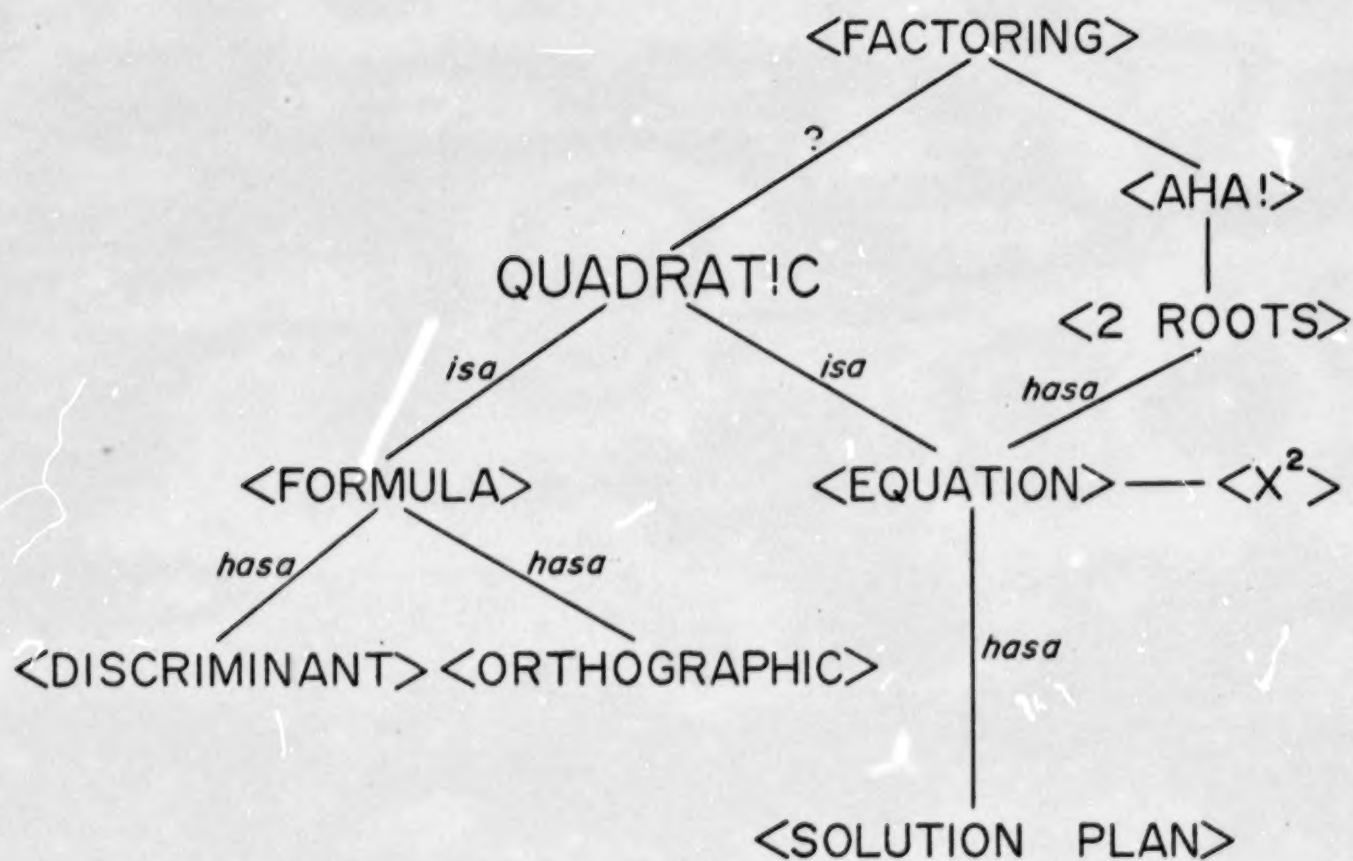


Figure 1



1. CAN FORMULA BE APPLIED?
2. PUT IN CANONICAL FORM.
3. GET VALUES OF COEFFICIENTS.
4. CALCULATE DISCRIMINANT.
5. IMAGINARY ROOT?
6. CALCULATE VALUE OF RADICAL.
7. NUMERATOR PLUS, ROOT 1.
8. NUMERATOR MINUS, ROOT 2.
9. STOP.

Figure 2

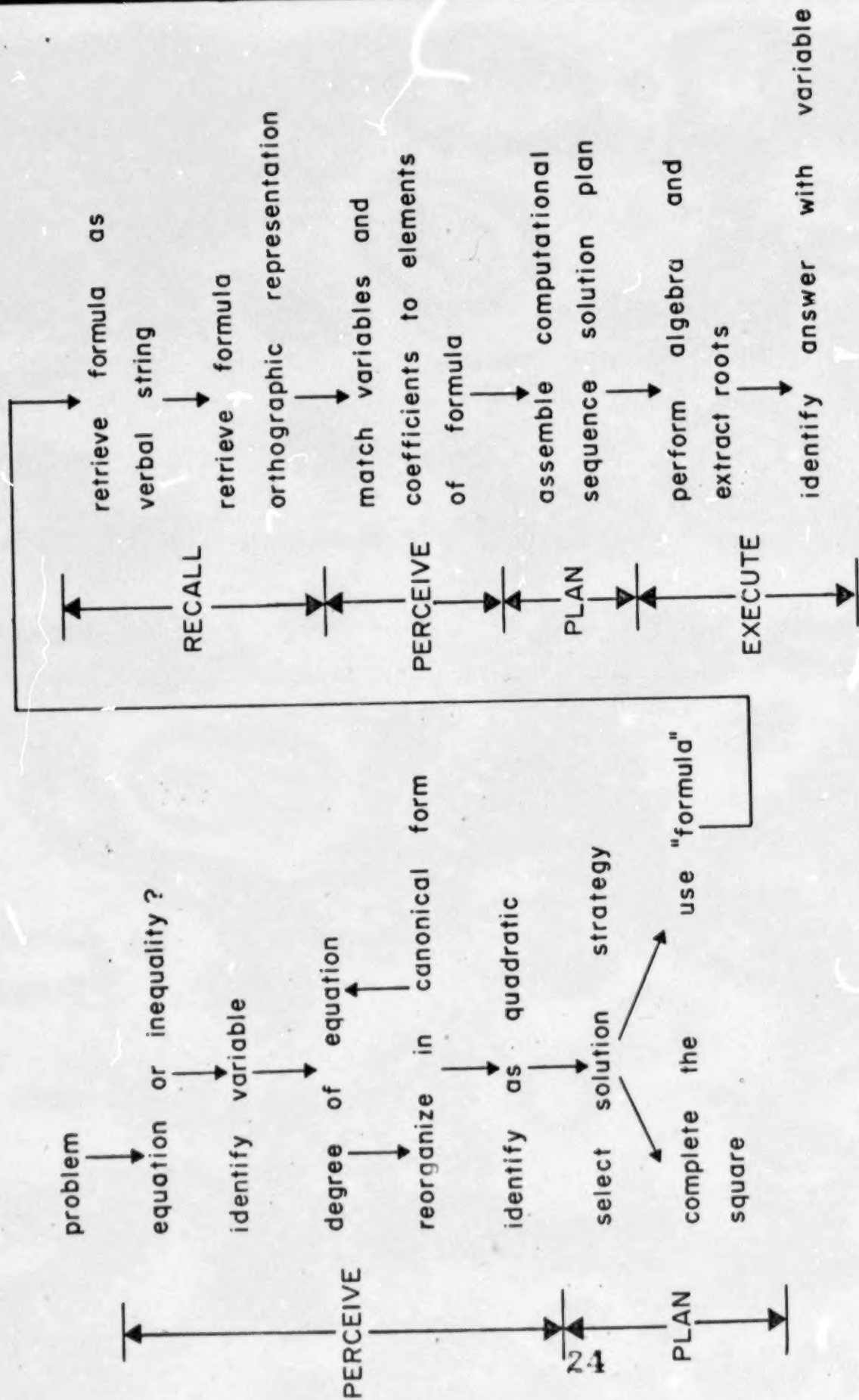


Figure 3